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HEAT TRANSFER DURING BUBBLING OF GAS THROUGH LIQUID

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Аннотация—Устройства, использующие барботаж газа через жидкость, широко распространены в промышленности. Изучению гидродинамики при барботаже газа через жидкость посвящено большое количество работ, однако теплообмен к стенкам аппарата или теплообменных элементов изучен менее полно. В данной работе рассматривается вопрос о теплообмене к стенке аппарата и змеевикам (трубы) в барботажных устройствах с нулевым расходом жидкои фазы (барботажных реакторах).

NOMENCLATURE

H_k,	height of gas–liquid bed;
d_k,	column diameter;
d_p,	diameter of heat-transfer element tube;
l_T,	mean amplitude of fluctuations;
R_b,	gas bubble radius;
S,	interface;
w_0'',	reduced gas velocity;
w'',	true gas velocity;
U_T'',	fluctuating gas velocity;
U_T,	fluctuating liquid velocity;
w_b,	velocity of gas bubble coming to the surface;
α_n,	mean heat-transfer coefficient determined by velocity fluctuations normal to surface;
α_p,	mean heat-transfer coefficient determined by velocity fluctuations tangential to surface;
$\bar{\alpha}$,	mean heat-transfer coefficient;
α_l.	heat-transfer coefficient at $\varphi \approx 1{\cdot}0$;
Nu,	$= \alpha l_T/\lambda$, Nusselt number;
Re,	$= U_T l_T/\nu$, characteristic turbulent Reynolds number;
Pr,	= Prandtl number;
φ,	volumetric gas content;
N_g,	work performed by gas;
N_T'',	work of turbulent gas fluctuations;
N_T,	work performed by turbulent liquid fluctuations;
ρU_T^2,	fluid phase shearing stresses;
γ'', γ,	gas and liquid specific weights;
ν,	kinematic viscosity of liquid;
μ_w,	dynamic viscosity of liquid near the wall;
σ,	surface tension;
λ,	heat conductivity of liquid;
g,	free fall acceleration;
$c_1, c_2, c_1', c_2', c_1'', c_2''$,	constants;
$\rho U_T''^2$,	gas phase shearing stresses;
μ,	dynamic viscosity of liquid.

THEORY

A STRICT theoretical solution of the problem of heat transfer to the surface of an apparatus or coil-pipe in devices with gas bubbling through liquid at zero flow rate of liquid phase, i.e. at zero mean velocity, offers great difficulties. In this case the heat-transfer rate is probably determined by the rate of turbulent fluctuations. Because of the complicated character of the hydrodynamic pattern, we shall use in this case the method proposed earlier by the present author for the analysis of heat transfer in installations with a mixer [2].

Due to non-linearity of the motion of gas bubbles the fluctuating motion of liquid is of three-dimensional nature. Assuming that the fluctuations in various directions take equal part in heat transfer, one can express the mean heat-transfer coefficient as follows

$$\bar{\alpha} = \tfrac{1}{3}\alpha_n + \tfrac{2}{3}\alpha_p \qquad (1)$$

For the surface in a flow normal to the wall, according to reference [1],

$$Nu_n = \frac{\alpha_n l_T}{\lambda} = 1{\cdot}14\, Re^{0{\cdot}5}\, Pr^{0{\cdot}37} \left(\frac{\mu}{\mu_w}\right)^{0{\cdot}14} \qquad (2)$$

For the surface in a flow parallel to the wall, according to references [1, 3],

$$Nu_p = \frac{\alpha_p l_T}{\lambda} = 0{\cdot}035\, Re^{0{\cdot}8}\, Pr^{\frac{1}{3}} \left(\frac{\mu}{\mu_w}\right)^{0{\cdot}14} \qquad (3)$$

Thus the problem turns into the determination of the characteristic size l_T and mean fluctuation velocity U_T.

When determining the heat transfer to the apparatus wall it is natural to take the value proportional to the diameter of the apparatus or the distance between the apparatus walls as the characteristic size

$$l_T = c d_k$$

Following Kutateladze [3] one can tentatively take the maximum scale of fluctuations as equal to

$$l_{T\max} \approx \tfrac{1}{3} d_k$$

and the minimum one

$$l_{T\min} \to 0$$

Then, tentatively, the mean size of fluctuations will be

$$l_T \approx \tfrac{1}{6} d_k \qquad (4)$$

In the case of heat transfer to a coil-pipe or a tube built into the apparatus the diameter of the coil-pipe should be taken as the characteristic size

$$l_T \approx d_p \qquad (5)$$

Proceeding from the above, the case of heat transfer to the apparatus wall and to the coil-pipe wall should be considered separately.

Determine the rate of fluctuations U_T. To this end consider the energy balance of a bubbling bed.

In bubbling of gas through liquid the work performed by steady-state gas on the path H_k is

$$N_g = \varphi \cdot \frac{\pi d_k^2}{4} \cdot H_k^2(\gamma - \gamma'') \qquad (6)$$

Since the kinetic energy of the gas bubbles coming to the surface is small, all the work of the gas goes into the turbulent fluctuations of the liquid and gas

$$N_g = N_T'' + N_T \qquad (7)$$

In the above balance the work of formation of an interface is also not taken into account, since the interface in steady state is constant.

The work of turbulent fluctuations of liquid for the surface element of the column is

$$dN_T = \rho U_T^2 \pi d_k \, dH \, . \, H \qquad (8)$$

where ρU_T^2 is the shearing stresses, $\pi d_k \, . \, dH$ the area of the element, H the path of fluctuations.

By integrating within the range from 0 to H_k, we obtain

$$N_T = \int_0^{H_k} dN_T = \tfrac{1}{2}\rho U_T^2 \pi d_k H_k^2. \qquad (8a)$$

The work of turbulent fluctuations of gas for a bed element is

$$N_T'' = \rho'' U_T''^2 H \, dS. \qquad (9)$$

Substituting the values of dS and integrating within the range from 0 to H_k we obtain

$$N_T'' = \int_0^{H_k} dN_T'' = \varphi \frac{3\rho''}{2R_n} U_T''^2 \frac{\pi d_k^2}{4} H_k^2 \qquad (9a)$$

To estimate the value of N_T as the mean radius of a bubble, the Levich formula [5] can be used

$$R_b = 6^{\frac{1}{3}} \frac{\sigma}{w''^2 (\rho'' \rho^2)^{\frac{1}{3}}} \qquad (10)$$

Further write equation (7) in the form

$$N_T = N_g\left(1 - \frac{N_T''}{N_g}\right)$$

and estimate the term

$$\frac{N_T''}{N_g}$$

It is evident that the rate of gas fluctuation velocity could not exceed the actual gas velocity, i.e.

$$U_T'' < w_0''/\varphi$$

Substituting the value of U_T'' into the expression for N_T'' we find that

$$\frac{N_T''}{N_g} \leqslant \frac{\gamma''}{\gamma - \gamma''} \cdot \frac{3w_0''^2}{2\varphi^2 g R_n} \tag{11}$$

By substituting numerical values in this expression it is not difficult to see that for low-pressure gas, mainly used for bubbling of gas through liquid, the relation is

$$\frac{N_T''}{N_g} \ll 1{\cdot}0$$

Thus, practically

$$N_g \approx N_T \tag{12}$$

However, when the pressure increases in the apparatus, i.e. with the increase of $[\gamma''/(\gamma - \gamma'')]$ and decrease of σ (i.e. R_n), the term $[1 - (N_T''/N_g)]$ should be taken into account.

Thus from (12) we find that

$$U_T \approx \varphi^{\frac{1}{2}}(\tfrac{1}{2}gd_k)^{\frac{1}{2}}\frac{\gamma}{\gamma - \gamma''} \tag{13}$$

For the determination of heat generation, substituting the value of U_T into (2) and (3) and adding according to (1), we obtain

$$Nu = \frac{\alpha l_T}{\lambda} = \tfrac{1}{3}1{\cdot}14\left(\frac{\varphi^{\frac{1}{2}}(\frac{1}{2}gd_k)^{\frac{1}{2}}\,l_T}{\nu}\right)^{0{\cdot}5} \times Pr^{\frac{1}{3}}\left(\frac{\mu}{\mu_w}\right)^{0{\cdot}14} + \tfrac{2}{3}0{\cdot}035\left(\frac{\varphi^{\frac{1}{2}}(\frac{1}{2}gd_k)^{\frac{1}{2}}\,l_T}{\nu}\right)^{0{\cdot}8} \times Pr^{\frac{1}{3}}\left(\frac{\mu}{\mu_w}\right)^{0{\cdot}14} \tag{14}$$

The double-term formula (14) can be represented graphically (Fig. 1) by plotting against the Reynolds number.

$$Re = \frac{\varphi^{\frac{1}{2}}(\frac{1}{2}gd_k)^{\frac{1}{2}}\,l_T}{\nu}$$

From Fig. 1 it follows that within the range

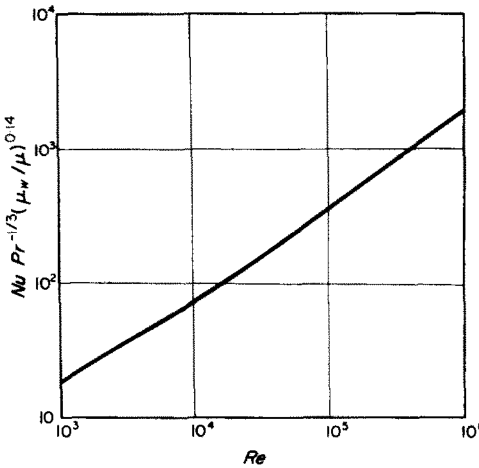

FIG. 1. *Nu* vs. *Re*,

$$Nu = \frac{\alpha l_T}{\lambda}; \qquad Re = \frac{\varphi^{\frac{1}{2}}(\frac{1}{2}gd_k)^{\frac{1}{2}}l_T}{\nu}.$$

of the numbers $Re = 10^3$–10^5 formula (14) can be presented as a single-term formula in the form

$$NuPr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0{\cdot}14} \approx 0{\cdot}2Re^{0{\cdot}67} \tag{15}$$

To find the particular relation between the numbers *Nu* and *Re* the characteristic size l_T should be substituted into their values.

(a) *Heat transfer to the apparatus wall*

When heat transfer proceeds to an apparatus wall, taking the characteristic size according to (4), we obtain

$$Nu = \frac{\alpha d_k}{\lambda} = c_1\varphi^{\frac{1}{3}}\left(\frac{gd_k^3}{\nu^2}\right)^{\frac{1}{3}} Pr^{\frac{1}{3}}\left(\frac{\mu}{\mu_w}\right)^{0{\cdot}14} \tag{16}$$

Since the heat-transfer coefficient is independent of the apparatus size, one can write

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} Pr^{-\frac{1}{3}} \left(\frac{\mu_w}{\mu}\right)^{0\cdot 14} = c_1\varphi^{\frac{1}{3}} \tag{17}$$

where

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}}$$

is the analogue of the Nusselt number often used in heat-transfer theory for condensation.

(b) *Heat transfer to a coil-pipe wall*

Taking the coil-pipe diameter as the characteristic size, we obtain

$$\frac{\alpha d_p}{\lambda} = c_2\varphi^{\frac{1}{3}}\left(\frac{g d_p^3}{\nu^2}\right)^{\frac{1}{3}}\left(\frac{d_k}{d_p}\right)^{\frac{1}{3}} Pr^{\frac{1}{3}}\left(\frac{\mu}{\mu_w}\right)^{0\cdot 14} \tag{18}$$

or similarly to the above

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} . Pr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0\cdot 14} = c_2\varphi^{\frac{1}{3}}\left(\frac{d_k}{d_p}\right)^{\frac{1}{3}} \tag{19}$$

Thus in this case the volumetric gas content φ is the only parameter characteristic on which heat-transfer rate depends.

To determine the relation between the gas content of a bed and the gas velocity as well as the physical properties, we shall use the results of experiments on gas content of a bubbling bed. The formula proposed by Kutateladze [4] is the simplest one and has been confirmed by various experiments.

Using the relation proposed by Kutateladze

$$\varphi = 0\cdot 4\left(w_0''\Big/\sqrt[4]{\left[\frac{g^2\sigma}{(\gamma-\gamma'')}\right]}\right)^{\frac{2}{3}}\left(\frac{\gamma''}{\gamma}\right)^{0\cdot 15} \tag{20}$$

we obtain

(a) for heat transfer to an apparatus wall

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} Pr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0\cdot 14} = c_1'\frac{w_0''^{0\cdot 22}}{\left(\dfrac{g^2\sigma}{\gamma-\gamma''}\right)^{0\cdot 055}}\left(\frac{\gamma''}{\gamma}\right)^{0\cdot 55} \tag{21}$$

(b) for heat transfer to a heat-transfer element

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} Pr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0\cdot 14} = c_2' w_0''^{0\cdot 22}\left(\frac{\gamma-\gamma''}{g^2\sigma}\right)^{0\cdot 055}\left(\frac{\gamma''}{\gamma}\right)^{0\cdot 05}\left(\frac{d_k}{d_p}\right)^{\frac{1}{3}} \ldots \tag{22}$$

It follows from (21) and (22) that the surface tension and the density of liquid and gas as well as the pressure, only slightly influence the heat-transfer coefficient, which fact is confirmed by numerous investigations.

Since from physical considerations the gas content of a bed cannot exceed 1, formulas (21) and (22) are applicable up to the values of velocities defined by equation (20) (at $\varphi = 1\cdot 0$). For $\varphi = 1$ the equations for the determination of the heat-transfer coefficient will be of the form:

(a) for heat transfer to the apparatus wall

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} Pr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0\cdot 14} = c_1'' \tag{21'}$$

(b) for heat transfer to a heat-transfer element

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} Pr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0\cdot 14} = c_2''(d_k/d_p)^{\frac{1}{3}} \tag{22'}$$

MORE PRECISE DEFINITION OF CALCULATION FORMULAE AND THEIR COMPARISON WITH EXPERIMENTAL DATA

The above expressions (17) and (19) describe the relations between all main values on which heat-transfer rate depends. The values of the constants in equations (17), (19), (21) and (22) can be determined by substituting for the characteristic size the values from equations (4) and (5). The calculated values are $c_1'' = 0\cdot 26$; $c_2'' = 0\cdot 187$.

However, because of the approximate character of the definition of the mean characteristic size, the values of the constants obtained by calculations or from experimental data should be compared. From the analysis of the experimental data the following values can be proposed for the constants

$$c_1' = 0{\cdot}25\text{–}0{\cdot}28; \qquad c_2' = 0{\cdot}18.$$

Substituting the values of the constants into equations (17) and (19) we finally obtain:

(a) for heat transfer to the apparatus wall

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} Pr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0{\cdot}14} = 0{\cdot}25\varphi^{\frac{1}{3}} \qquad (23)$$

(b) for heat transfer to the coil-pipe surface

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} Pr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0{\cdot}14} = 0{\cdot}18\varphi^{\frac{1}{3}}\left(\frac{d_k}{d_p}\right)^{\frac{1}{3}} \qquad (24)$$

To make convenient comparison with the experimental data, we present the calculation formulae for heat-transfer determination as follows, neglecting the variation of the physical constants σ, γ and γ'':

(a) heat transfer to an apparatus wall

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} Pr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0{\cdot}14} 0{\cdot}19 w_0''^{0{\cdot}22} \qquad (25)$$

(b) heat transfer to the wall of a heat-transfer element

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} Pr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0{\cdot}14} = 0{\cdot}14 w_0''^{0{\cdot}22}\left(\frac{d_k}{d_p}\right)^{0{\cdot}33} \qquad (26)$$

The comparison of the calculation expression with the experimental data by Kolbel [8] and Fair [9] is presented in Figs. 2 and 3, from which it follows that the experimental and predicted data agree quite satisfactorily.

Formula (26) agrees satisfactorily with the data by Sokolov and Salomakhin [6], [7] who proposed the relation*

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right) Pr^{-\frac{1}{3}} = 0{\cdot}3\left(\frac{w_0''}{w_d}\right)^{0{\cdot}2} \qquad (27)$$

The comparison of expressions (26) and (27) for $d_k/d_p = 7\text{–}6$ shows that they are also very close.

From (24) on substituting $\varphi = 1$, we can obtain the limit value of the heat-transfer coefficient α_l, which is

$$\frac{\alpha_l}{\lambda}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} Pr^{-\frac{1}{3}}\left(\frac{\mu_w}{\mu}\right)^{0{\cdot}14} \approx 0{\cdot}32$$

According to the experimental data studied by Sokolov and Salomakhin

$$\frac{\alpha_l}{\lambda}\left(\frac{\nu^2}{g}\right) Pr^{-\frac{1}{3}} \approx 0{\cdot}23\text{–}0{\cdot}32$$

i.e. the predicted and experimental values of α_l are also very close.

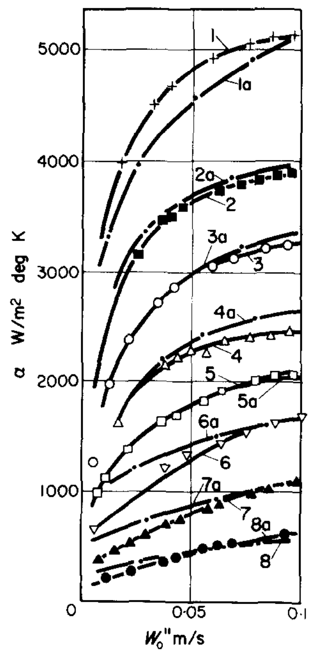

FIG. 2. Experimental data compared with the predicted ones of Kolbel [8] by formula (24) for a liquid with varying viscosity (MN s/m²).

1. —$\mu \approx 0{\cdot}9$;	2. —$\mu = 1{\cdot}23\text{–}1{\cdot}76$;
3. —$\mu = 2{\cdot}3\text{–}3$;	4. —$\mu = 4{\cdot}45\text{–}6{\cdot}75$;
5. —$\mu = 11\text{–}16$;	6. —$\mu = 26\text{–}29$;
7. —$\mu = 77\text{–}118$;	8. —$\mu = 736\text{–}945$.

(subscript a means calculation).

CONCLUSIONS

An approximate theoretical analysis has been carried out for the problem of heat transfer to the heating surface with bubbling of gas

* In the present work the physical properties were determined by the mean temperature of the bed near the wall.

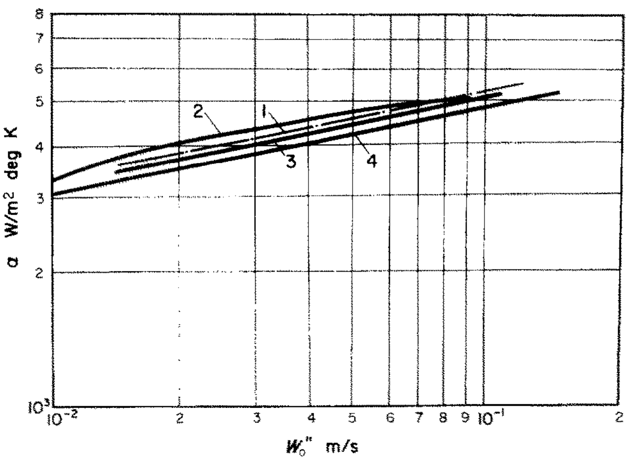

FIG. 3. Experimental results compared with predicted data. (Water–air system.)

1. —data of Fair [9];
2. —data of Kolbel [8];
3. —data of Sokolov [6];
4. —equation (24).

through the liquid, and which is based on the application of the equations of energy balance in a bubbling bed to the medium volume under consideration. As a result of the analysis, it was found within the accuracy of the constant that the heat-transfer coefficient depended on the regime parameters, geometric sizes of the apparatus (heat-transfer element) and on the physico-chemical properties of the medium.

The predicted values of the constant are close to that obtained from the analysis of the experimental data.

Design formulae are recommended.

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Abstract—Devices which use bubbling of gas through liquid are widely used in industry. A great number of works have dealt with the hydrodynamics of bubbling of gas through liquid, however heat transfer to the walls of heat-transfer installation has been neglected. The present work studies heat transfer to the walls of an apparatus and coil-pipe in bubbling devices with zero flow rate of liquid phase (bubbling reactors).

Zusammenfassung—Vorrichtungen zum Durchleiten vor Gasblasen durch Flüssigkeiten sind in der Industrie weit verbreitet. Eine grosse Zahl von Arbeiten hat die Hydrodynamik des Durchsatzes von Gas durch Flüssigkeiten behandelt, doch wurde der Wärmeübergang an die Wände der Wärmeübertragungsapparatur vernachlässigt. Die vorliegende Arbeit untersucht den Wärmeübergang an die Wände der Apparatur und die Rohrwendel von Blaseneinrichtungen beim Flüssigkeitsdurchsatz Null (Blasenreaktoren).